Problem Set 5

It's OK to work together on problem sets.

1. Consider a pure exchange economy under uncertainty composed of a number of households. There are three types of households, A, B, and C. There are three states of the world, 1, 2, and 3. There is a single consumption good which is deliverable in each of the three states in differing amounts. The households receive perfectly correlated random endowments of the form:

Type A: 50 units of the consumption good, if state 1 occurs, 100 units if state 2 occurs, 300 units if state 3 occurs.

Types B and C: 200 units if state 1 occurs, 400 units if state 2 occurs, 1200 units if state 3 occurs.

Type A households are risk neutral, and believe that the three states of nature will occur with equal subjective probability. Their subjective utility a random consumption bundle {C1 if state 1 occurs, C2 if state 2 occurs, C3 if state 3 occurs} is given by $U^A(C1, C2, C3) = C1+C2+C3$.

Type B households believe that states 1 and 2 are impossible. Their subjective utility of a random consumption bundle {C1 if state 1 occurs, C2 if state 2 occurs, C3 if state 3 occurs} is given by $U^B(C1, C2, C3) = C3$.

Type C households are infinitely risk averse (with positive subjective probability of each state occurring) and their subjective utility of a random consumption bundle {C1 if state 1 occurs, C2 if state 2 occurs, C3 if state 3 occurs} is given by $U^C(C1, C2, C3) = \min [C1, C2, C3]$.

Agents sell all of their endowment as contingent commodities at prevailing prices and buy any nonnegative portfolio of contingent commodities they wish. No short selling is allowed. Consider a population consisting of <u>four</u> households of type A, one of type B, and one of type C.

(i) We propose as competitive equilibrium prices for the three state-contingent commodities $p^*=(1/3, 1/3, 1/3)$. Demonstrate that these are competitive equilibrium prices by deriving the competitive equilibrium consumption bundles for each of the three types of agents and then demonstrating that markets clear.

(ii) Now suppose that there are large numbers of agents in the economy: 400 type A, 100 type B, 100 type C. How do competitive equilibrium prices change? Explain.

(iii) The First Fundamental Theorem of Welfare Economics (Starr's *General Equilibrium Theory: An Introduction*, Theorem 12.1) still applies, but it needs interpretation. Describe the efficiency properties of the competitive equilibrium allocation. Is everyone pleased with his choice (ex ante) while they are holding a portfolio of contingent commodities before the uncertainty is resolved? Is everyone pleased with his choice (ex post) after the uncertainty is resolved?

2. In discussing the relationship of saving to consumption in a monetary economy, J. M. Keynes writes

"An act of individual saving means --- so to speak --- a decision not to have dinner to-day. But it does *not* necessitate a decision to have dinner or to buy a pair of boots a week hence or a year hence or to consume any specified thing at any specified date. Thus ... [saving] depresses the business of preparing to-day's dinner without stimulating the business of making ready for some future act of consumption...If saving consisted not merely in abstaining from present consumption but in placing simultaneously a specific order for future consumption, the effect might indeed be different."

--- J. M. Keynes, *The General Theory...*, chap. 16.

Consider the budget constraint of a household in an Arrow-Debreu economy with a full set of futures markets without uncertainty. Can the difficulty Keynes notes ("[saving] depresses the business of preparing to-day's consumption without stimulating ... some future act of consumption") occur in an Arrow-Debreu economy with a full set of futures markets in equilibrium? Explain.

3. Consider a voting plan for a group of voters ranking ten possibilities for group decisionmaking: A, B, C, D, E, F, G, H, I, J. Each voter submits a ballot ranking the possibilities. The voting procedure then gives his first place choice a weight of 10; the second place choice is given a weight of 9; ...; the tenth place choice is given a weight of 1. For each possibility, the weighted votes of all the voters are then added up. The possibility achieving the highest total of weighted votes is declared the winner.

When the group needs to choose among a subset of possibilities, the full ballot is reviewed and the ranking on elements of the subset is based on the weights that come from the full ranking.

Evaluate the weighted voting procedure in terms of the Sen version of the Arrow axioms. Does the procedure fulfill

a. Pareto Principle?
b. Independence of Irrelevant Alternatives?
c. Non-Dictatorship?
d. Unrestricted Domain?
e. Will voters find it advantageous to misstate their true preferences to influence the outcome?

4. To solve the problem of cyclic majorities in pairwise voting, suppose we adopt the following procedure:

All of the propositions to be ranked are divided into pairs (with possibly an odd proposition left uncontested) and a majority vote is taken on each pair separately. The winners are then grouped into pairs and the process repeated. This process continues until there are just two propositions remaining. A majority vote is taken on them and the winning proposition is thereby chosen.

Demonstrate by example that

(i) the choice under this rule may depend on the pattern of pairing at the first (and later) stages of the process (this is known as 'agenda manipulation').

(ii) some voters may find it advantageous to misstate their preferences in the early voting to influence the choices available in later voting.

The observations (i) and (ii) reflect failures of the voting procedure to fulfill some of the Arrow properties : Pareto Principle, Independence of Irrelevant Alternatives, Non-Dictatorship, Unrestricted Domain. Explain which Arrow principles are not fulfilled and how that leads to (i) and (ii).

5. Consider an Arrow-Debreu economy with a full set of contingent commodity markets under uncertainty with equilibrium prices and allocations. After a few periods of time have passed, some uncertainty has been resolved.

(i) Give sufficient conditions so that, if markets reopen for trade, relative prices for the remaining contingent commodities will be unchanged from the original equilibrium and there will be no active trade.

(ii) Give sufficient conditions so that, if markets reopen for trade, relative prices for the remaining contingent commodities will change from the original equilibrium and there will be active trade.

(iii) Explain.